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LEVEL OF ADAPTABILITY OF FIVE CONIC MAP PROJECTION VARIANTS ON MACEDONIAN NATIONAL AREA AS STATE MAP PROJECTION

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SUMMARY

Nowadays a large number of cartographic projections are used for many purposes; some of them are utilized as cartographic state projections. In the Republic of Macedonia the Gauss-Krüger projection is in use as official state cartographic projection. Regarding the position of the territory and the criteria that the state projection has to fulfill, the Gauss-Krüger projection is not the most favorable projection for projecting the state territory. The goal of this paper is the comparison of the state projection with all variants of conformal cone projections while analyzing the results. From the obtained results, it is defined that the average linear deformation in the first variant is **1.98 cm/km**, in the second variant **is 1.96 cm/km**, in the third variant **2.61 cm/km**, in the fourth variant **1.78 cm/km** and in the fifth variant is **2.95 cm/km**. These values are smaller when compared to the average linear deformation of Gauss-Krüger projection that is **8.49 cm/km**. The conformal cone projections enable symmetrical dispersion of the isograms, and the maximal linear deformation is **8.49 cm/km**. While with the Gauss-Krüger projection the maximal value of the linear deformation is **25.4 cm/km**.

Key words: state cartographic projection, cone projection, deformations, linear scale, isograms.

INTRODUCTION

The Republic of Macedonia is located in the central part of the western Balkans (Figure.1.), and it spreads between parallels with latitudes from 40^o to 42^o, while the distance from the Greenwich meridian is around 22^o. The geographical coordinates of the most extreme points including the central point are given in Figure.2. The Republic of Macedonia has an area of 25

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713 km^2 , while the length of the bordering line is 899 km; these parameters show that this is a small state that has the shape of ellipse.

Figure. 1. Location of the R.M.

Point	Geographic coordinates					
	φ			λ		
North	42°	22′	21″	22°	18′	4″
South	40°	51′	16″	21°	7′	33″
East	41°	42′	33″	23°	2′	12″
West	41°	31′	4″	20°	27′	32″
Center	41°	35′	0″	21°	45′	0″

Figure.2. Extreme points of the R.M.

The Gauss-Krüger projection is used as the state cartographical projection in the R.M. and it was established in the 1924 year, when the R.M was part of the SFRY regime. The central meridian in use is the meridian with the geographical longitude of 21 ° that passes through the western part of the territory. This causes a part of the territory to be projected with a higher linear deformation from the permitted (10cm/km), and the dispersion of the isocholes to be asymmetrical regarding the territory (Figure.4.). Linear deformation will not pass the value of 10 cm/km and the dispersion of the isocholes will be symmetrical if we use two coordinate systems (Figure.3.). Considering the size of R.M and the need for transformation of the coordinates in the areas that are located in the border of the two coordinate systems the use of two coordinate systems is unacceptable.

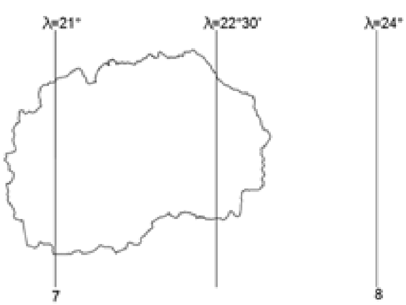

Figure.3.R.M in two zones of Gauss-Kruger projection

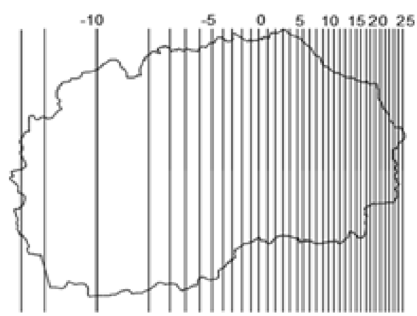

Figure.4.Isocholes in Gauss-Kruger projection

EFFORTS FOR NEW STATE CARTOGRAPHIC PROJECTION OF THE REPUBLIC OF MACEDONIA

Gauss-Krüger projection does not fulfill the criteria for projecting all of the teritory with deformations smaller than 10 cm/km and symmetrical dispersion of the isograms regarding the territory. As a result a lot of reseaeches have been made trying to define varinats of projections that can be applied as state cartographical projection. The main factors taken into account when choosing the state cartographical projection are (Shehu. A., 1971):

- The value of the maximal linear deformation
- The dispersion of the linear deformation
- The adjustment of the mathematical model for geodetic calculations.

Taking into consideration the fact that the R.M lies along parallels and it is located in middle geographical latitude, the Lambert conformal conic projections would be a good variant to use as state cartographic projection. The aim of this research is the comparison between all variants of the conformal cone projections with the Gauss-Krüger projection and the possibility of implementing conformal cone projections as state cartographic projection. The objectives of this research are:

- Suitability of conformal cone projection for the territory of R.M.
- Analyzing the value of deformations
- Dispersions of deformations
- Implementation of cone projections as official state projection.

For this specific purpose a test-model was selected, that consists of 38 points located across the territory (Figure.5.). From the total number 33 points are represented by cities in the R.M while four points are the most extreme ones, including the central point. In conformal cone projections the deformations depend on the geographical latitude so the distribution of the points in this test model is suitable for this criteria, also a lot of geodetic duties with high precision are made in cities for which the value of the deformations is necessary.

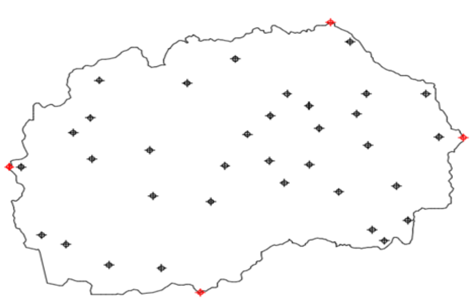

Figure. 5. Dispersion of the points from the test model

For all points of the test model, the linear deformations are calculated in all possible variants of the conformal cone projections and from them the average linear deformations in a proper variety are conducted. The results are compared with the average linear deformation of the Gauss-Krüger projection that is calculated with the implementsation of the same test model.

$$\theta = \frac{\sum_{1}^{n} |\Delta d|}{n} \tag{1}$$

where: θ - average value of linear deformation

Δd - linear deformation

n - number of points from test model.

CONFORMAL CONE PROJECTION

Conformal cone projections have been primarily developed by Lambert (Snyder. J.P., 1987). These projections enable the projection of the earth's ellipsoid in the cone shape through particular mathematical models, most applicable for state cartographic projection are the conformal cone projections in which the tangent cone (Figure.6.) or the secant cone can be used (Figure.7.).

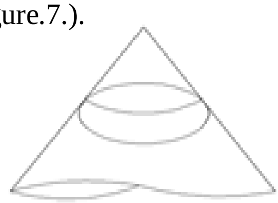

Figure. 6. Tangent cone

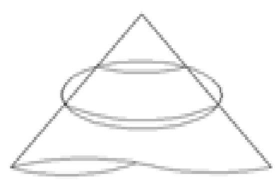

Figure. 7. Secant cone

The determination of the position of the points in the two dimensional coordinate system is enabled through the use of the polar and the rectangular coordinate system (Figure.8.).

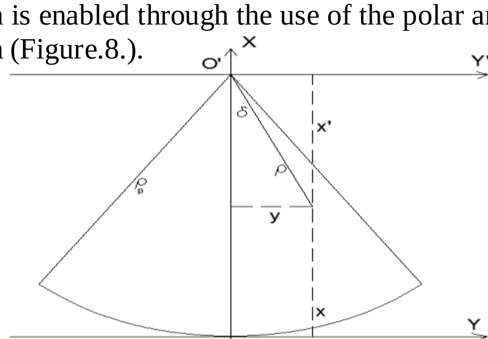

Figure.8. The rectangular and polar coordinative system in conformal cone projections

The relationship between the rectangular coordinates and the polar coordinates in the projection is enabled through the following formulas (Frančula, N., 2004):

$$y = \rho \sin \delta \tag{2}$$

$$x = \rho_p - \rho \cos \delta . \tag{3}$$

One of the most significant features of the conformal cone projections is that there are no deformation of the angles ($w=0$) that causes the linear scale in the direction of the meridian and the parallels to have equal value *($m=n$)*.

When processing the variants of cone projections for the Republic of Macedonia the ellipsoid of Bessel (Bessel 1841) is used. This ellipsoid is also used in the processing of the Gauss-Krüger projection. The Bessel ellipsoid is still in use nowadays in the R.M. as the official ellipsoid.

I – VARIANT

This variant of the conformal cone projections requires the use of the tangent cone where the linear deformations are positive, meaning that the length in the map is longer regarding the same length in the ellipsoid. In this variety when projecting, the following condition is required:

- The parallel with the given longitude (φ_o) needs to be projected without deformations *($n_0=1$)*.

Regarding this condition the determination of the constant of the projection *“k”* and the constant of the integration *“K”* are conducted through the following formulas (Srbinoski. Z., 2012):

$$k = \sin \varphi_o \tag{4}$$

$$K = \frac{r_o U_o^k}{k} . \tag{5}$$

As a parallel that will be projected without deformations is the parallel that passes through the central point of the R.M, apart from it in the north and the south the deformations rise. As the Y - axis of the rectangular coordinative system it is considered the tangent of the parallel that passes through the point with the minimal geographic latitude (Figure.2.), while the X - axis is a projection of the meridian that has the geographic longitude of 21 ^{o} .In order to avoid the use of the negative coordinates through the Y- axis, the Y coordinate starts with a value of 50 000 m. From the conducted calculations it is estimated that the average linear deformation for all points from the test model is **1.98 cm/km**, the maximal linear deformation at the northern point is **9.49 cm/km**, while the central point is projected without deformations.

The city of *Radovis* has the minimal linear deformation of **0.01 cm/km** while the city of *Kriva Palanka* has the maximal linear deformation of **5.85 cm/km**. The dispersion of the isograms is symmetrical in relation to the central point and the territory, its graphical presentation is shown in Figure.9.

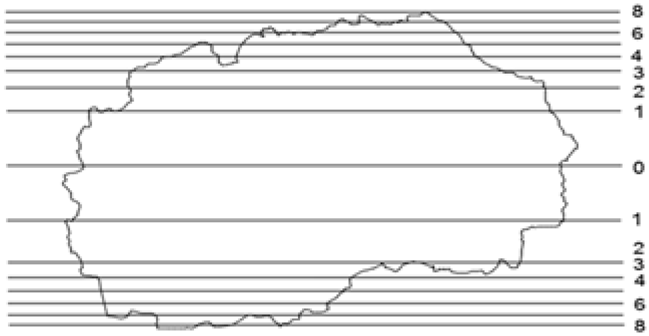

Figure.9. Isograms in the first variant

II-VARIANT

In this variant the tangent cone is also used, and the cone touches the ellipsoid in the parallel that passes through the middle of the Republic of Macedonia, meaning that the geographical latitude of this parallel is determined as the average value from the geographical latitude of northern and southern point, so the geographical latitude of this parallel is $f_o=41^o36'48.5''$.

In this variant during the projection of the ellipsoid the next requirements should be met:

- The parallel that pass in the middle of the territory (f_o) should be projected without deformations $(n_o=1)$
- The linear scale in the bordering parallels should be equivalent$(n_S=n_N)$.

Regarding these two conditions the constants *"k"* and *"K"* are determined with the following formulas (Srbinoski. Z., 2009):

$$k=\frac{\log r_S-\log r_N}{\log U_N-\log U_S} \quad (6)$$

$$K=\frac{r_o U_o^k}{k}. \quad (7)$$

The coordinate system in all variants is defined as in the first variety. From the conducted calculations it is estimated that the average linear deformation has a value of ***1.96 cm/km***. The maximal linear deformation appears in the bordering parallels and has a value of ***8.74 cm/km***, while the minimal linear deformation appears in the standard parallel (f_o) of ***0.00 cm/km***. From the cities, *Radovis* has the minimal linear deformation of ***0.00 cm/km,*** while the city of *Kriva Palanka* has the maximal linear deformation of ***5.27 cm/km***. The dispersion of the isograms is symmetrical in relation to the standard parallel. Its graphical presentation is shown in Figure.10.

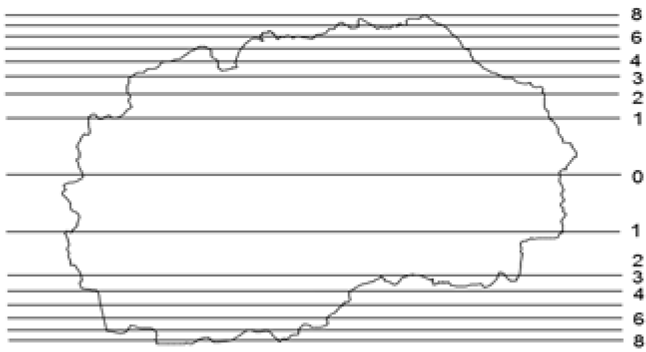

Figure.10. Isograms in second variant

III-VARIANT

This variant foresees the projecting of the ellipsoid in the secant cone in which the following condition should be met:

- Two parallels with the given geographical latitude (standard parallels) should be projected without deformations *($n_1=n_2=1$).*

According to the majority of scientists best results are obtained if these two standard parallels are postured in 1/6 and 5/6 from the length of the arch of the meridian through the bordering parallels. For the territory of the Republic of Macedonia these parallels reach the geographical latitudes $f_1=41^o06'25''$ and $f_2=42^o07'09''$.Taking into consideration this condition the determination of the constant of the projection *"k"* and the constant of the integration *"K"* are conducted through the next formulas (Srbinoski. Z., 2009):

$$k = \frac{\log r_1 - \log r_2}{\log U_2 - \log U_1} \tag{8}$$

$$K = \frac{r_1 U_1^k}{k} = \frac{r_2 U_2^k}{k}. \tag{9}$$

From the conducted calculations it is defined that the absolute value of the average linear deformation measures ***2.61 cm/km***. The maximal linear deformation occurs in the northern point and has a value of ***4.88 cm/km***. The parallels with the given geographical latitude are projected without deformations. From the cities, the city of *Ohrid* has the minimal linear deformation of ***-0.01 cm/km,*** while the city of *Radovis* has the maximal linear deformation of ***-3.89 cmk/km***. The dispersion of the isograms is symmetrical in relation to the standard parallels. Its graphical presentation is shown in Figure.11.

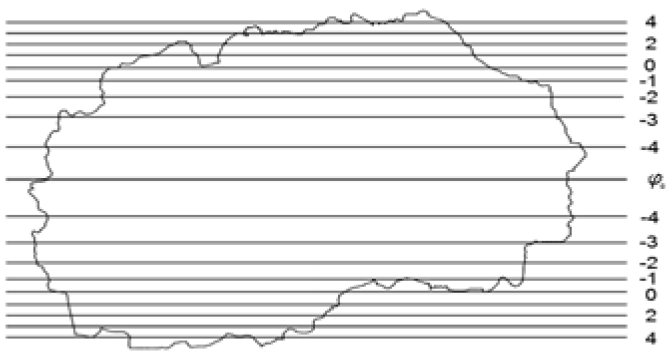

Figure. 11. Isograms in the third variant

IV-VARIANT

In the fourth variant the projection of the ellipsoid is done in the secant cone, in which the following conditions should be met:

- The standards parallels should be projected without deformations (n_1=1).
- The bordering parallels should have the same deformation (n_S=n_N).

Taking into consideration these conditions the determination of the constant of the projection *"k"* and the constant of the integration *"K"* are calculated through the following formulas (Srbinoski. Z., 2009):

$$k = \frac{\log r_S - \log r_N}{\log U_N - \log U_S} \tag{10}$$

$$K = \frac{r_1 U_1^k}{k}. \tag{11}$$

The parallel with the geographical latitude of $f_1 = 42^0 00'00''$ is adopted as a standard parallel, while the geographical latitude of the second parallel is estimated with iterative procedure, and its geographical latitude is $f_1 = 41^0 13' 45.1''$.

From the conducted calculations it is defined that the absolute value of the average linear deformation is ***1.78 cm/km***. The maximal linear deformation occurs in the southern point and has a value of ***-6.49 cm/km***, while the standard parallels are projected without deformations. From the cities, the city of *Dojran* has the minimal linear deformation of ***-0.08 cm/km,*** while the city of *Kriva Palanka* has maximal linear deformation of ***3.02 cm/km***. The dispersion of the isograms is symmetrical regarding to the standard parallels and its graphical presentation is shown in Figure.12.

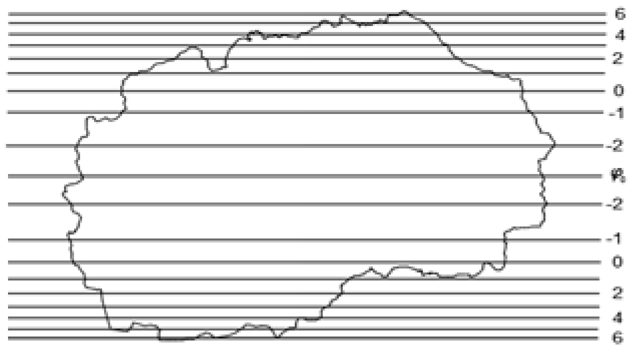

Figure.12. Isograms in the fourth variant

V-VARIANT

This variant requires the projection of the ellipsoid to be done in the secant cone, in which the following conditions should be met:

- The bordering parallels of the territory that is projected should have the same deformations.
- The largest scale should be larger from 1, for as much as the smallest scale is smaller from 1.

Taking into consideration these conditions the constant of the projection *"k"* and the constant of the integration *"K"* are determined through the following formulas (Srbinoski. Z., 2012):

$$k = \frac{\log r_S - \log r_N}{\log U_N - \log U_S} \tag{12}$$

$$K = \frac{2 r_N U_N^k r_o U_o^k}{k(r_n U_N^k + r_o U_o^k)} \tag{13}$$

$$\varphi_o = \arcsin(k) \tag{14}$$

where:
r_s- the radius of the point that lies on the southern parallel
r_n- the radius of the point that lies on the northern parallel
r_o- the radius of the parallel with the minimal linear scale
f_o- the geographical latitude of the parallel with the minimal linear scale.
From the conducted calculations it is established that the absolute value of the average linear deformation is ***2.95 cm/km***. The maximal linear deformation occurs in the most extreme points in the north and south, and has a value of ***4.37 cm/km***. The same value of the deformation occurs in the city of *Radovis* with ***-4.37 cm/km***, while the smallest deformation occurs in the city of *Resen* with ***-0.06 cm/km***. The dispersion of the isograms is graphically presented in Figure.13, from which it can be seen that dispersion of isochloes is symmetrical in relation to the standard parallels.

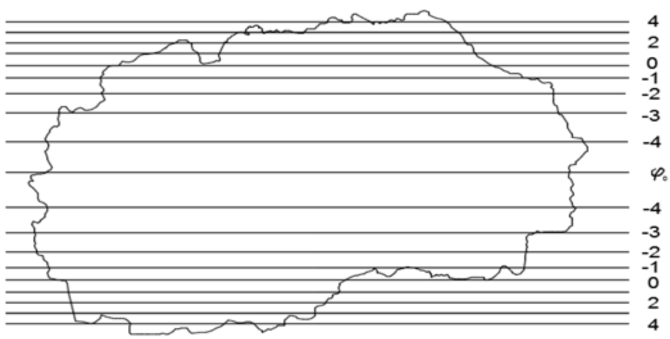

Figure.13.Isograms in the fifth variant

CONCLUSIONS

The conformal cone projections have a major application in cartography; approximately 90 % of the projections that are used around the world apply the Lambert conformal cone projection or the Transversal Mercator projection (Iliffe.J and Lott.R., 2008). Conformal cone projections offer a good solution for projecting the territory that lies in the medium latitudes and along parallels. From the results that are shown in Table.1 it is confirmed that the use of any variant of conformal cone projections would be much favourable than the Gauss-Krüger projection that is in use as official state cartographical projection.

Table 1. Results from conformal cone projections and Gauss-Krüger projections conducted from test model

Significant factors	*Lambert conform conical projection*					*Gauss-Krüge projection*
	I	*II*	*III*	*IV*	*V*	
Θ	***1.98***	***1.96***	***2.61***	***1.78***	***2.95***	***8.49***
m_{max}(cm/km)	***9.49***	***8.74***	***4.88***	***6.49***	***4.37***	***25.4***
Dispertion of deformations (cm/km)	***0 to 9.49***	***0 to 8.74***	***-3.89 to 4.88***	***-2.25 to 6.49***	***-4.37 to 4.37***	***-10 to25.4***

It is confirmed that equal values are obtained using either *I* or *II* variant, in which the tangent cone is applied. Precisely in the second variant a smaller linear deformation appears because the standard parallel passes through the middle of the territory of Republic of Macedonia. From the other three variants in which the secant cone is applied, the *IV* variant gives a better chance of projecting because in this variant smaller value of average linear deformation are obtained. The value of average linear deformation in *IV* variant is smaller in comparison with the all other variants.
In Gauss-Krüger projection the criteria for projecting the territory with deformations up to 10 cm/km is not reached in the eastern part of the R.M. Roughly around 12% of our territory is projected with deformations higher than 10 cm/km, also this projection has asymmetry in the dispersion of the isograms. From the conducted research it is concluded that the conformal cone projections are suitable for the territory of R.M., all variants offer smaller deformation and symmetrical dispersion of deformations. From the five processed variants, the *IV* variant would be the most favorable variant, but its implementation as state cartographic projection would induce a series of additional issues. The actual state cartographic projection is being used for a long period of time and a lot of geodetic activities; for example geodetic maps and topographic maps are developed in the Gauss-Krüger projection, so the change of the projection would also result in solving the issues that would occur.

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