

DETERMINING POINTS STABILITY IN GEODETIC CONTROL NETWORKS USING HANNOVER METHOD

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SUMMARY

The structures and the land which they are built on, are under constant and / or occasional action of external and / or internal forces, which leads to geometric deformation and their displacement. For the purpose of timely detection of the mentioned deformations and displacements, it is necessary to perform continuous control measurements. Such control measurements can be implemented through certain geodetic methods. By using geodetic methods, information on structure or ground displacement and deformation can be obtained. This paper presents the general procedure of determining the stability of points using a Hannover method and its mathematical procedure. The practical part of this paper, carried out by Hannover method is based on measurements in two epochs (1.Epoch 96 and 2.Epoch 97) carried out in purpose of testing and calibration of certain methods and instruments at the polygon „Novoselka“ (Vuchkov, 2000). Application of the method in this paper aims to present the procedure for its performance and determine its strength by comparison of achieved results with the results of other method applied on the same measurements. In order to preserve continuity in the paper, there is a brief explanation of the need and way of setting the geodetic control network, elevation of the measurements and part of the world mostly applied methods of examination of points stability. The Conclusion also refers to the deficiency of all statistical methods in general, due to the duration of time needed to perform the geodetic measurements.

Key words: geodetic control network, discrete point, displacements, deformations.

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1. GEODETIC CONTROL NETWORK

Geodetic networks that are established for the needs of engineering geodesy are called special purpose networks, whereas networks that are used to determine displacements and deformations of structures are called geodetic control networks. As in all geodetic problems, as well as in the tasks for determining the displacements and / or deformations of certain objects, the procedure is performed through a certain number of points called discrete points of the object. The discrete points are used to analyze eventual displacement or deformations, whereas basic points are certain number of points located outside the unstable area, but still close to the object, through which the discrete points of the object are observed.

In order to be able to control the object's stability at all through the discrete points embedded in it, it is necessary to connect them to the points of the basic network in a single coordinate system both in position and height. This set of points defined in a single coordinate system is called a geodetic control network (hereinafter referred to as GCN)

The shape and size of the network depends of the shape and dimensions of the object, the configuration of the terrain and the expected deformations and / or displacements of the object. The number of points in the basic network should be as low as possible but not less than 4 points, while the number of discrete points depends on the size of the object and the expected deformations and / or displacements. GCN are most often presented in the Cartesian rectangular coordinate system in:

- one dimensional (1D) for height position;
- two dimensional (2D) for plane position and / or a combination of
- three dimensional (3D) coordinate system for the position of the grid points in the space.

The relative position of the points from the GCN is determined with geodetic measurements. In order to achieve the basic goal for which the network is established, i.e. even a small value of the displacement vector to be detected, the network should be precise and reliable.

Measurements are performed in time differences so-called epochs, which usually depend on the speed of expected displacements and / or deformations of the object.

The first epoch, also known as the zero epoch, is established after the stabilization of the GCN itself, while the remaining ones are carried out successively in the period of time determined by the basic project for the object.

2. EQUALIZATION OF GCN

As mentioned before, geodetic measurements that primarily serve to determine the relative position of points in the GCN are specified with an a priori analysis project. The method of data processing and the determination of the unknown parameters that define the position of the points in the GCN, depends on the type of measurements that were performed.

The most commonly used method of equalization of measured data is the method of indirect equalization. GCN are adjusted as free networks with minimal trace in all points of the basic network for the first epoch, while in the following epochs of the measurements the equalization is completed with minimal trace in the points of the basic network, which will be determined to be stable or not.

After analyzing, removing gross errors using Data snooping, and completed equalization, evaluation vectors ($\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2$) and correspondent singular cofactor matrices ($\mathbf{Q}_{\hat{\mathbf{x}}_1}, \mathbf{Q}_{\hat{\mathbf{x}}_2}$) about the points from GCN are obtained (in the previous and in the current measurement epoch). The vector $\hat{\mathbf{d}}$ and the matrix $\mathbf{Q}_{\hat{\mathbf{d}}}$, are basic for performing deformation analysis and are calculated using the following equations:

$$\hat{\mathbf{d}} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1 \quad \dots(1)$$

$$\mathbf{Q}_{\hat{\mathbf{d}}} = \mathbf{Q}_{\hat{\mathbf{x}}_2} + \mathbf{Q}_{\hat{\mathbf{x}}_1} \quad \dots(2)$$

3. METHODS FOR POINTS STABILITY ANALYSIS

In order to examine the stability of points in the GCN, a number of methods have been developed that most had received their names after the research centers and / or the authors (Nasevski, 2001).

The most famous world methods can be listed:

- **Hannover** (Pelzer, 1971),
- **Delft** (Baarda, 1968, Kok, 1977),
- **Munich** (Chrzanowski, 1981),
- **Fredericton** (Chrzanowski et al., 1982) etc.

These methods are basically static methods. With the application of static methods, it is assumed that due to the short period of time in which the measurements were performed, there were no deformation and / or displacement of the object and / or the ground on which the points of the GCN were stabilized.

4. HANNOVER METHOD

This paper presents the application of the *Hannover method* or the Hannover procedure for examining the stability of points in the GCN. This method was developed by Pelzer (1971) and (1974), and its practical application was adapted by Nimeier (1976) and (1982) year.

Like most other methods, this method is based on examination of the displacements through the difference of the coordinates of the points by conducting a global stability test calculated as the mean discrepancy between two consecutive measurements (epochs) in the GCN.

4.1. MATHEMATIC MODEL

4.1.1 ANALYZING THE HOMOGENEITY OF TWO MEASUREMENT EPOCHS

In order to be able to determine the stability of the points through the difference in the coordinates in the GCN obtained in the procedure of equalization, it is necessary to analyze the homogeneity of the measurements in both epochs (Nasevski, 2001; Pelzer, 1971; Mihailović and Aleksić 1994, 2008; Ašanin, 2003; Vrce, 2011).

Therefore two hypotheses are set:

$$H_0: \mathbf{M}(s_1^2) = \mathbf{M}(s_2^2) - \text{нулта хипотеза наспроти} \quad \dots(3)$$

$$H_a: \mathbf{M}(s_1^2) \neq \mathbf{M}(s_2^2) - \text{алтернативна хипотеза} \quad \dots(4)$$

Statistic test:

$$T \left| H_0 = \frac{s_1^2}{s_2^2} \right| H_0 \sim F_{(f_1, f_2)}, \text{ каде } s_1^2 > s_2^2 \quad \dots(5)$$

$$T \left| H_0 = \frac{s_2^2}{s_1^2} \right| H_0 \sim F_{(f_1, f_2)}, \text{ каде } s_2^2 > s_1^2 \quad \dots(6)$$

- If $T < F_{1-\alpha, f_1, f_2} \Rightarrow H_0$ the measurements in both epochs are with homogeneous accuracy. The unified dispersion factor is calculated the following equation:

$$s^2 = \frac{f_1 s_1^2 + f_2 s_2^2}{f_1 + f_2}, \quad \dots(7)$$

where, f_1 and f_2 are degrees of freedom

- If $T > F_{1-\alpha, f_1, f_2} \Rightarrow H_a$, the measurements in both epochs are not with homogeneous accuracy. In this case the unified dispersion factor (s^2) is not calculated, and homogenization of the measurements needs to be done.

4.1.2 GLOBAL TEST FOR POINTS STABILITY OF GCN

The stability of the points implies there were no displacement in the time period between the two eras, i.e. a stable point is point that kept its position between two sets of measurements. To perform the test two hypotheses were set:

$$H_0: \mathbf{M}(\hat{\mathbf{x}}_1) = \mathbf{M}(\hat{\mathbf{x}}_2) - \text{zero hypothesis} \quad \dots(8)$$

$$H_a: \mathbf{M}(\hat{\mathbf{x}}_1) \neq \mathbf{M}(\hat{\mathbf{x}}_2) - \text{alternative hypothesis} \quad \dots(9)$$

where the parameter $\hat{\mathbf{d}}$ is calculated using the equation (1), after which the **gap** or the secondary discrepancy is calculated according to the formula:

$$\theta^2 = \frac{\hat{\mathbf{d}}^T \mathbf{Q}_d^+ \hat{\mathbf{d}}}{h} \quad \dots(10)$$

Where:

$$h = \text{rang}(\mathbf{Q}_d)$$

$$\mathbf{Q}_d^+ = \mathbf{P}_d$$

Statistic test:

$$\begin{aligned} T \Big|_{H_0 = \frac{\theta^2}{S^2}} H_0 &\sim F_{h,f}, \\ T \Big|_{H_0 = \frac{\theta^2}{S^2}} H_0 &\sim F_{h,f,\lambda} \end{aligned} \quad \dots(11)$$

$$\lambda = \frac{1}{\sigma^2} (\mathbf{Hx}_a - \mathbf{w})^T - (\mathbf{HQ}_{\hat{\mathbf{x}}}\mathbf{H}^T)(\mathbf{Hx}_a - \mathbf{w}), \quad \dots(12)$$

λ – non-centrality parameter.

- If $T < F_{1-\alpha, h, \lambda} \Rightarrow H_0$ Points are not displaced with probability of $(1 - \alpha)$,
- If $T > F_{1-\alpha, h, \lambda} \Rightarrow H_a$ There is at least one displaced point

With this test, global information about network stability in two different epochs is obtained.

4.1.3 GLOBAL TEST FOR POINTS STABILITY FROM BASIC GEODETIC NETWORK

As previously stated, GCN consists of a set of points from the basic geodetic network and a set of points of the object (discrete points).

For this purpose, the following hypotheses come together:

$$H_0: \mathbf{M}(\hat{\mathbf{x}}_{s_1}) = \mathbf{M}(\hat{\mathbf{x}}_{s_2}) - \text{zero hypothesis} \quad \dots(13)$$

$$H_a: \mathbf{M}(\hat{\mathbf{x}}_{s_1}) \neq \mathbf{M}(\hat{\mathbf{x}}_{s_2}) - \text{alternative hypothesis} \quad \dots(14)$$

where:

$\hat{\mathbf{x}}_{s_1}$ - evaluation vector for the coordinates from the previous epoch

$\hat{\mathbf{x}}_{s_2}$ - evaluation vector for the coordinates from the current epoch.

In order to examine the stability of the points of the basic network, the vector of the coordinate differences is divided into two sub-factors:

- $\hat{\mathbf{d}}_s$ – for basic points

- $\hat{\mathbf{d}}_o$ – for discrete points

$$\hat{\mathbf{d}} = \begin{bmatrix} \hat{\mathbf{d}}_s \\ \hat{\mathbf{d}}_o \end{bmatrix}, \quad \dots(15)$$

This procedure also means dividing the matrix of weights into a submatrix of weights in the following way:

$$\mathbf{P}_{\hat{\mathbf{d}}} = \begin{bmatrix} \mathbf{P}_{ss} & \mathbf{P}_{so} \\ \mathbf{P}_{os} & \mathbf{P}_{oo} \end{bmatrix} \quad \dots (16)$$

The square form is presented with two independent square sub forms. The first refers to the mismatch of the basic points, and the second to the mismatch of the points of the object:

$$\hat{\mathbf{d}}^T \mathbf{P}_{\hat{\mathbf{d}}} \hat{\mathbf{d}} = \hat{\mathbf{d}}_s^T \bar{\mathbf{P}}_{ss} \hat{\mathbf{d}}_s + \bar{\mathbf{d}}_o^T \mathbf{P}_{oo} \bar{\mathbf{d}}_o \quad \dots(17)$$

Where:

$$\bar{\mathbf{d}}_o = \hat{\mathbf{d}}_o + \mathbf{P}_{oo}^{-1} \mathbf{P}_{os} \hat{\mathbf{d}}_s, \quad \dots(18)$$

$$\bar{\mathbf{P}}_{ss} = \mathbf{P}_{ss} - \mathbf{P}_{so} \mathbf{P}_{oo}^{-1} \mathbf{P}_{os} \quad \dots(19)$$

Using the equation (10) the average discrepancy or gap is calculated:

$$\theta_s^2 = \frac{\hat{\mathbf{d}}_s^T \bar{\mathbf{P}}_{SS} \hat{\mathbf{d}}_s}{h_s}, \quad \dots(20)$$

Where: $h_s = \text{rang } \bar{\mathbf{P}}_{SS}$.

Statistic test:

$$T \mid H_0 = \frac{\theta_s^2}{S^2} \mid H_0 \sim F_{h,f}, \quad \dots(21)$$

According the equation (21): If $T < F_{1-\alpha, h, f}$, than H_0 is accepted. If $T > F_{1-\alpha, h, f}$, than H_a is accepted.

4.1.4 LOCATING DISPLACED POINTS IN THE BASIC GEODETIC NETWORK

When the global test shows the existence of displaced points in the basic network, the displaced points must be located. Therefore the coordinate vector of the points in the basic network is divided into two sub-factors:

- $\hat{\mathbf{d}}_F$ - which contains the difference of the points coordinates which are conditionally stable, and
- $\hat{\mathbf{d}}_B$ - which contains the difference of the points coordinates that are considered as unstable.

$$\hat{\mathbf{d}}_s = \begin{bmatrix} \hat{\mathbf{d}}_F \\ \hat{\mathbf{d}}_B \end{bmatrix}, \quad \dots(22)$$

The next step is division the cofactor matrix into the next submatrix:

$$\mathbf{P}_{SS} = \begin{bmatrix} \mathbf{P}_{FF} & \mathbf{P}_{FB} \\ \mathbf{P}_{BF} & \mathbf{P}_{BB} \end{bmatrix}, \quad \dots(23)$$

The square form is presented with two independent square sub forms:

$$\hat{\mathbf{d}}_s^T \bar{\mathbf{P}}_{SS} \hat{\mathbf{d}}_s = \hat{\mathbf{d}}_F^T \bar{\mathbf{P}}_{FF} \hat{\mathbf{d}}_F + \hat{\mathbf{d}}_B^T \mathbf{P}_{BB} \hat{\mathbf{d}}_B \quad \dots(24)$$

Where:

$$\bar{\mathbf{d}}_B = \hat{\mathbf{d}}_B + \mathbf{P}_{BB}^{-1} \mathbf{P}_{BF} \hat{\mathbf{d}}_F, \quad \dots(25)$$

$$\bar{\mathbf{P}}_{FF} = \mathbf{P}_{FF} - \mathbf{P}_{FB} \mathbf{P}_{BB}^{-1} \mathbf{P}_{BF} \quad \dots(26)$$

For every point from the basic geodetic network an average discrepancy is calculated:

$$\theta_j^2 = \frac{\bar{d}_B^T \mathbf{P}_{BB} \bar{d}_B}{h_B}, \quad \text{za } j = 1, 2, \dots, k. \quad \dots(27)$$

$h_B = \text{rang } \mathbf{P}_{BB}$ ($h_B = 2$, for two-dimensional network).

In the set of k – points, the point with maximum value θ_j^2 is recognized and the point that responds to θ_{max}^2 it is said to be displaced and it is removed from the basic geodetic network.

For the remaining $k-1$ points an average discrepancy is calculated:

$$\theta_{REST}^2 = \frac{\bar{d}_F^T \mathbf{P}_{FF} \bar{d}_F}{h_s - 2}, \quad \dots(28)$$

Followed by the statistic test:

$$T \mid H_0 = \frac{\theta_{REST}^2}{S^2} \mid H_0 \sim F_{h, 2, f}. \quad \dots(29)$$

If $T < F_{1-\alpha, h, f}$, then H_0 is accepted and we conclude there are no displaced points. If $T > F_{1-\alpha, h, f}$, then H_a is accepted. If this is the case, the procedure is repeated until the test (29) shows that there are no displaced points in the network.

After the procedure, the points in the basic network are divided into displaced and unmodified. In further analysis of the GCN, displaced points from the basic network are treated just like the points of the object.

5. ANALYSIS OF THE POINTS STABILITY OF THE GEOPOLYGON "NOVOSELKA" WITH THE APPLICATION OF THE HANNOVER METHOD

For this practical procedure, a test of stability of GCN of the geopolygon "Novoselka" was carried out on the territory of municipality of Novo Selo (Vučkov, 2000).

The geopolygon as GCN consists of 21 points of which: 11 points of the basic network and 10 points of the object - the dam. In this procedure, the points from the basic network in the GCM are analyzed, and they serve as a geopolygon for the examination of other methods and calibration of geodetic instruments. The points of the basic network are stabilized in a geologically stable field, they are numbered with Arabic numerals from 1 to 11 and represent reinforced concrete pillars on which a forced centering system is

installed, with the exception of point No. 11 on the platform and it is stabilized with a metal wedge with dimensions of 10 mm x 100 mm. The test is only for the points of the basic network (Fig. 1) with an adopted plan of measurements of routes and lengths.

The general data for the core network are the following:

- $m = 11$ - Number of points
- $D_{min} = 87$ - Minimal length
- $D_{max} = 2082$ - Maximal length
- $\bar{D} = 611$ - Average length

The measurements were performed in 1996 and 1997 with duration of (Vučkov, 2000):

- Epoch 96 – seventeen consecutive days
- Epoch 97 – fourteen consecutive days

Table: 1: Specifications about the basic network in two different epochs (Vučkov 2000).

	Epoch 96	Epoch 97	
$n_{directions}$	85	87	Number of measured directions
$n_{lengths}$	35	44	Number of measured lengths
n	120	131	Total number of measurements
$\hat{\sigma}_0^2$	0.289''	0.424''	Dispersion coefficient of equalization
f	90	101	Degrees of freedom
σ_0^2	0.64''	0.64''	Dispersion coefficient a-priori

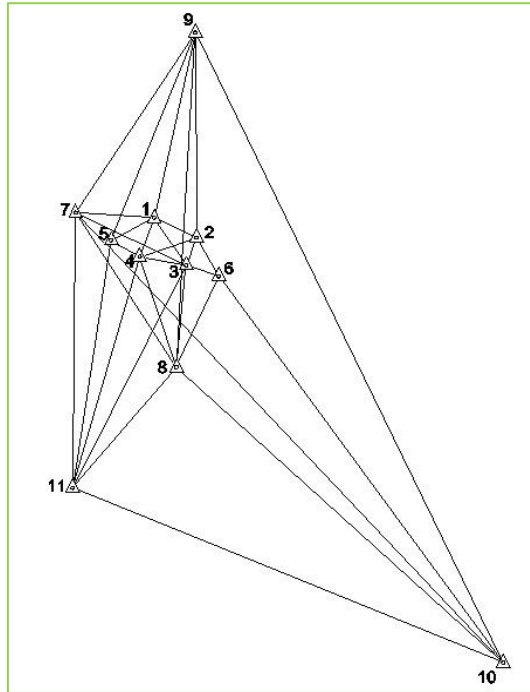


Fig.1: Geodetic network in the Geopolygon “Novoselka”

Table 2: Definitive coordinates (Nasevski, 2001)

Point	1. Epoch 96		2. Epoch 97		Difference [mm]	
	Y(m)	X(m)	Y(m)	X(m)	d_y	d_x
1	877.2302	1059.6939	877.2294	1059.6945	-0.8	0.6
2	999.9983	1000.0037	999.9982	1000.0032	-0.1	-0.5
3	969.9750	918.2695	969.9748	918.2689	-0.2	-0.6
4	835.7537	943.1784	835.7538	943.1796	0.1	1.2
5	752.2227	992.2113	752.2224	992.2120	-0.3	0.7
6	1064.0471	885.0298	1064.0478	885.0315	0.7	1.7
7	650.5019	1074.5252	650.5014	1074.5256	-0.5	0.4
8	941.3904	613.2868	941.3961	613.2851	-4.3	-1.7
9	996.1649	1612.4060	996.1667	1612.4084	1.8	2.4
10	1885.4414	-269.6264	1885.4447	-269.6286	3.3	2.2
11	642.0835	252.8102	642.0837	252.8080	0.2	-2.2

First of all, an analysis of the homogeneity of measurements in both epochs was made whose characteristics are given in Table 1 for Epoch 96 and

Epoch 97 based on hypothesys (3 and 4) and statistics test (5 and 6), the following results were obtained:

$$T = \frac{s_2^2}{s_1^2} = 1.467 < 1.617 = F_{0.99}(90,101),$$

therefrom, the hypothesis H_0 Is accepted, meaning that measurements in both epochs are with homogeneous accuracy and the dispersion coefficient $\sigma_0^2 = 0.64$ is accepted. The definite values of positional coordinates of the points obtained on the basis of previously carried out mediate equalization with minimal trace in all points of the basic network are given in Table 2. The global test (21) upon the previously calculated gap based on (20):

$$\theta_s^2 = \frac{\mathbf{d}_s^T \bar{\mathbf{P}}_{ss} \mathbf{d}_s}{h_{ss}} = \frac{60.744}{19} = 3.199,$$

$$T = \frac{\theta_s^2}{\sigma_1^2} = 4,999 > 1.641 = F_{0.95}(19,191),$$

it was concluded that in this set of points, there are one or more displaced points.

After this conclusion, the localization of the displaced points was reached, where five iterative procedures were carried out. It has been confirmed that points **8, 6, 4, 9** and **10** are displaced. By removing these points, the statistic test showed lower value of the quantile and the zero hypotheses was accepted, meaning there are no displacements for the other points of the basic network. The results are given in the Table 4.

Table 3: Short preview of the results of shifted points determined by the Hannover method in five interactive procedures.

Number	k	α_k	h_s	Shifted point	$T(H)$	$F_{1-\alpha, h_s, f}$
(0)	11	0.05	19		4.999	1.641
1	11	0.05	17	8	3.192	1.677
2	10	0.04556	15	6	2.826	1.744
3	9	0.04110	13	4	2.302	1.829
4	8	0.03661	11	9	1.961	1.940
5	7	0.03211	9	10	1.717	2.091

The results from the the carried our procedure for the same points by using the Munich method, undertaken from Nasevski, 2001, for the purpose of analysis of the both methods, are given in the Table 5.

Table 4: Short preview of the results of shifted points determined by the Munich method in four interactive procedures (Nasevski, 2001).

Number	m'	$\alpha_{m'}$	h_s	Shifted point	$T(H)$	$F_{1-\alpha_{m'},h_s,f}$
(0)	11	0.05	19		4.999	1.641
1	10	0.04556	17	10	5.075	1.644
2	9	0.04110	15	9	5.238	1.715
3	8	0.03662	13	6	4.960	1.804
5	7	0.03211	11	8	1.009	1.921

6. CONCLUSIONS

The deformation analysis procedure using the geodetic methods is an extensive and serious work that requires special attention. The Hannover method is a commonly accepted method for implementing such processes, due to its simplicity and high transparency of processes in the procedure up to the end result. Comparatively corresponding results obtained by the practical part of the paper and ones obtained by the method of Munich of the test polygon “Novoselka” (Nasevski, 2001) using the same measurements, define this method as acceptable and adequate for such procedures. Based on the results give in Table 3 and Table 4, it is clear that the both applied methods have identified identical points as unstable, as the points 10, 9, 6 and 8, and the difference is located in point 4, where it is shifted according to the Hannover method, whilst it is stable according to the Munich method.

The main disadvantage of this, and of all static methods is the time period of performing the measurements, in which we assume that deformations have not occurred. In the analyzed GCN for the this paper, the time period of measurement in both epochs is in average 15 days, during which there may not and should not ignore the fact that some deformation occurred as a consequence of some internal or external forces which affect the ground on which the points are stabilized. Such deficiency of static methods, and thus of this method, puts them in a subordinate role compared to dynamic methods. Therefore, when applying static methods, greater attention should be paid to the duration of measurements in order to avoid possible deformations occurring during measurement.

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